Morphological-Symmetry-Equivariant Heterogeneous Graph Neural Network for Robotic Dynamics Learning

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Introduction & Contributions

We propose **MS-HGNN**, a symmetry-equivariant heterogeneous graph neural network for robotic dynamics learning. By embedding kinematic structure and morphological symmetry into the model, MS-HGNN improves interpretability, data efficiency, and model efficiency. We validate our design through theory and experiments on diverse robotic tasks.

Preliminaries

Morphology-Informed HGNN. We use a Heterogeneous Graph Neural Network (HGNN), where *node and edge types reflect the robot's kinematic structure*. Nodes such as base, joints, and feet are modeled as distinct types ($\mathcal{V}_b$, $\mathcal{V}_t$, $\mathcal{V}_f$), while edges encode physical links ($\mathcal{E}_{ij}$) between them. This structured graph improves representation of rigid-body dynamics.

Morphological Symmetry. Rigid-body systems often exhibit symmetric kinematic branches and motions. We define a symmetry group $\mathcal{G}$ acting on robot states $(\mathbf{q}, \dot{\mathbf{q}})$, with:

$$g \overset{\diamond}{\triangleright} \mathbf{q} := \begin{bmatrix} \mathbf{X}_g \mathbf{X}_B \mathbf{X}_g^{-1} \\ \rho_{\mathcal{M}}(g)\mathbf{q}_{js} \end{bmatrix}, \quad g \overset{\diamond}{\triangleright} \dot{\mathbf{q}} := \begin{bmatrix} \mathbf{X}_g \dot{\mathbf{X}}_B \mathbf{X}_g^{-1} \\ \rho_{\mathcal{T}_\mathbf{q}\mathcal{M}}(g)\dot{\mathbf{q}}_{js} \end{bmatrix}$$

where $\mathbf{X}_B \in \mathbb{SE}_d$ is the base pose, and $\rho(g)$ is a group representation acting on the joint states. These symmetries are embedded in the GNN enabling weight sharing, enabling efficient learning and generalization across symmetric morphologies.

Methods

1. Determine the morphological symmetry group $\mathbb{G}_m < \mathbb{G}_\mathbb{E}$ and the unique kinematic branches $\mathbb{S}$ of the system, where $\mathbb{G}_\mathbb{E}$ is the generalized euclidean group.

2. Create subgraphs for all kinematic branches as $\mathcal{G}_i = \{\mathcal{G}_{i,1}(\mathbb{S}_{i,1}), \ldots, \mathcal{G}_{i,n_{\text{rep}}(\mathbb{S})}(\mathbb{S}_{i,n_{\text{rep}}(\mathbb{S}_i)})\}$, where $\mathcal{G}_{i,j_1} \cong \mathcal{G}_{i,j_2}, \forall j_1, j_2 \in \mathbb{N} \leq n_{\text{rep}}(\mathbb{S}_i)$.

3. Label each subgraph $\mathcal{G}_{i,j}$ as $\mathcal{G}_{p,q}$, where $p \leq |\mathbb{G}_m|$ corresponds to an element in group $\mathbb{G}_m$, and subgraphs with same q lies in the same orbit.

4. For any subgraph class $\{\mathcal{G}_q\}$, including the base node $\{\mathcal{V}_b\}$ that lacks the full set of $|\mathbb{G}_m|$ graphs, complete each group orbit by replicating elements along missing transformations and label them as $\mathcal{G}_{p,q}$.

5. Connect $\{\mathcal{V}_{b,p}\}$ with Cayley Graph. Connect each subgraph $\mathcal{G}_{p,q}$ to $\mathcal{V}_{b,p}$ with edge type $\mathcal{E}_q$, formalizing a graph $\mathcal{G}$.

6. Add input encoders and output decoders for each node based on the subgraph class p it belongs to, ensuring morphological symmetry equivariance $\mathbb{G}_m$ in our GNN.

MS-HGNN: Morphological-Symmetry-Equivariant Heterogeneous Graph Neural Network

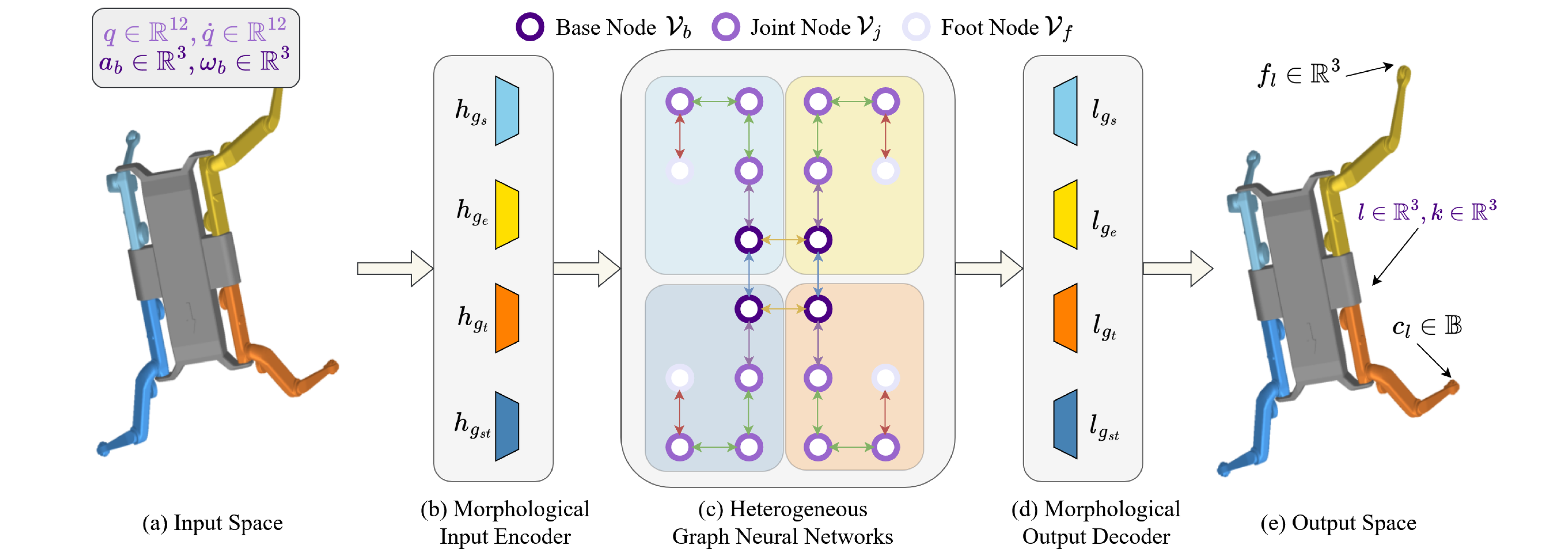

Figure 1: Overview of the MS-HGNN framework for robots with symmetry type $\mathbb{G} := \mathbb{K}_4$. (a) The input space consists of the robot's current state observations, which are mapped to corresponding nodes in the HGNN. (b) and (d) The morphological symmetry encoder-decoder pair ensures that the learned representations respect the robot's morphology. (c) The HGNN is automatically constructed to preserve geometric symmetry. (e) The output space consists of dynamics-relevant variables, obtained from their corresponding nodes in the HGNN.

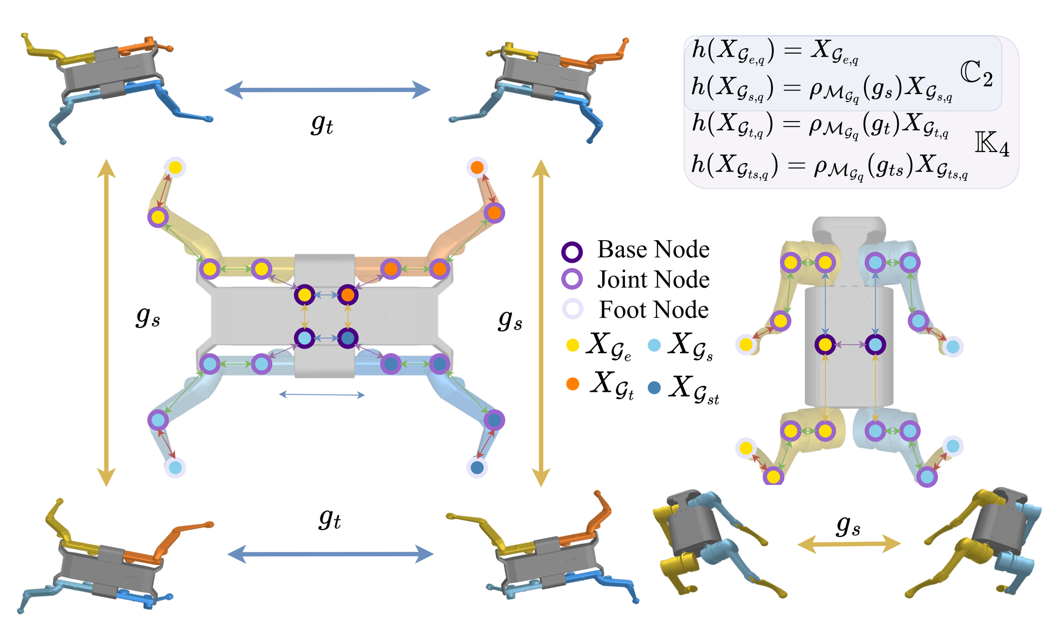

Figure 2: Morphological symmetry groups $\mathbb{G} := \mathbb{K}_4$ (left, Solo robot) and $\mathbb{G} := \mathbb{C}_2$ (right, A1 robot).

We prove that our constructed graph $\mathcal{G}$ is **equivariant** under **morphological symmetry transformations**.

- $\mathcal{G}$ is composed of subgraphs $\{\mathcal{G}_1, \ldots, \mathcal{G}_q\}$ (e.g., legs, arms), each with p symmetric instances.
- Symmetries are defined via group actions g_m:
- Euclidean: $g_m \triangleright \mathcal{G}_{p,q} = \mathcal{G}_{g_m(p),q}$
- Morphological: $g_m \overset{\diamond}{\triangleright} \mathcal{G}_{p,q} = \mathcal{G}_{g_m(p),q}$

We define the group action via permutation matrix ρ_b.

Theorem (Permutation Automorphism). If ϕ_{ρ_b} satisfies:

$$\rho_b A_\mathcal{G} \rho_b^T = A_\mathcal{G}, \quad \rho_b X_\mathcal{G} = X_\mathcal{G},$$

then it is a graph automorphism. **Lemma (Equivariance).** If ϕ_{ρ_b} is an automorphism, then:

$$g_m \triangleright z_\mathcal{G}(X_\mathcal{G}) = z_\mathcal{G}(g_m \triangleright X_\mathcal{G})$$

Theorem (MS-HGNN Equivariance). If encoder h and decoder l satisfy:

$$h(X) = \rho(g)X, \quad l(X) = \rho(g)^{-1}X,$$

then our HGNN satisfies:

$$g_m \overset{\diamond}{\triangleright} f_\mathcal{G}(X) = f_\mathcal{G}(g_m \overset{\diamond}{\triangleright} X)$$

Conclusion: Our MS-HGNN is **equivariant to morphological symmetry** and **generalizable** across robotic systems.

Contact State Detection (Classification) Experiment

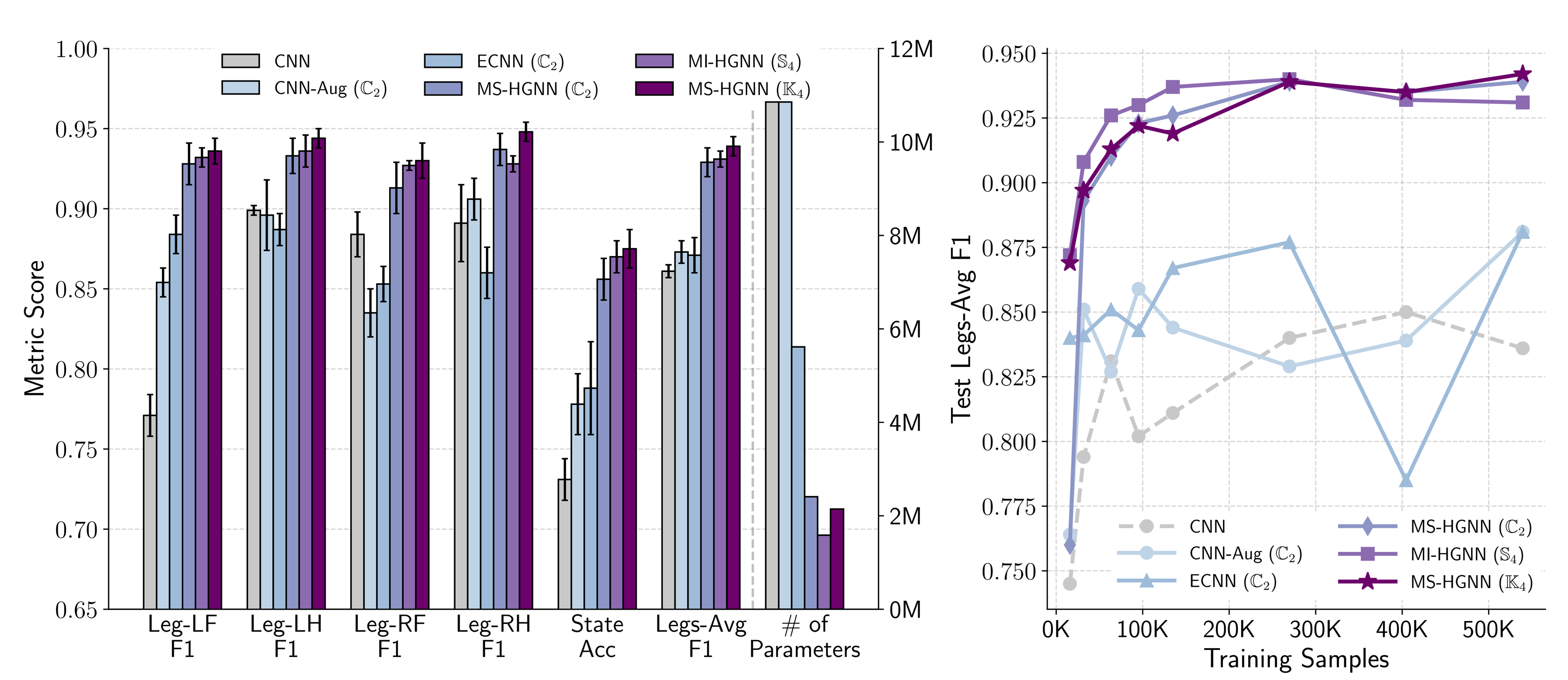

Figure 3: **Left:** Contact state detection and **Right:** Sample efficiency on the **real-world Mini-Cheetah** dataset [1].

Task: Predict 4-leg contact states from proprioceptive histories.

- Best Model: MS-HGNN-$\mathbb{K}_4$ (F1: 0.939, Acc: 0.875).
- Model Efficiency: Uses only 38% of ECNN's parameters.
- Sample Efficiency: Achieves ~0.9 F1 with 5% training data.

Conclusion: Symmetry-aware graph design improves accuracy, and parameter & data efficiency.

Ground Reaction Force Estimation (Regression) Experiment

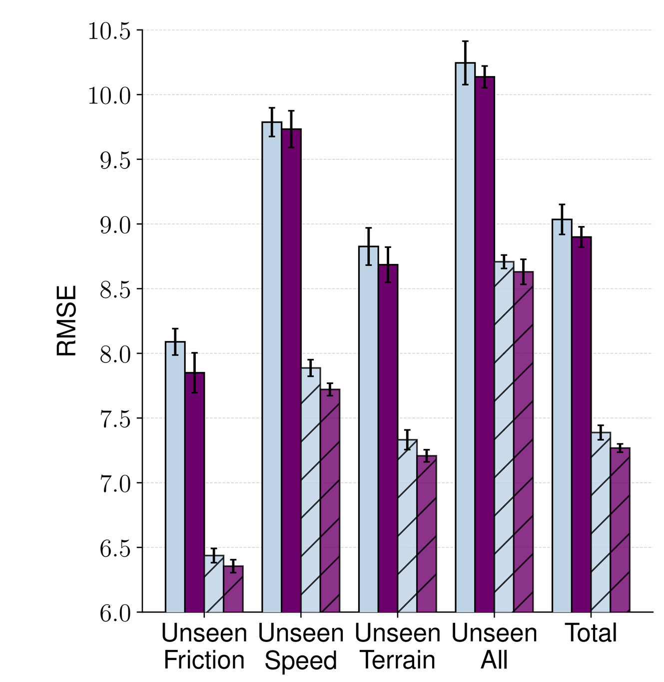

Figure 4: Ground reaction force estimation test RMSE on **simulated A1** dataset [2].

Task: Predict 1D/3D GRFs from 150-step proprioceptive history.

- MS-HGNN-$\mathbb{C}_2$ achieves the lowest RMSE, improving over MI-HGNN by 1.50% (1D) and 1.62% (3D).
- The relatively modest gain in 3D is because of the low X/Y GRF magnitudes.

Conclusion: Preserving symmetry improves force prediction across diverse terrains.

Centroidal Momentum Estimation (Regression) Experiment

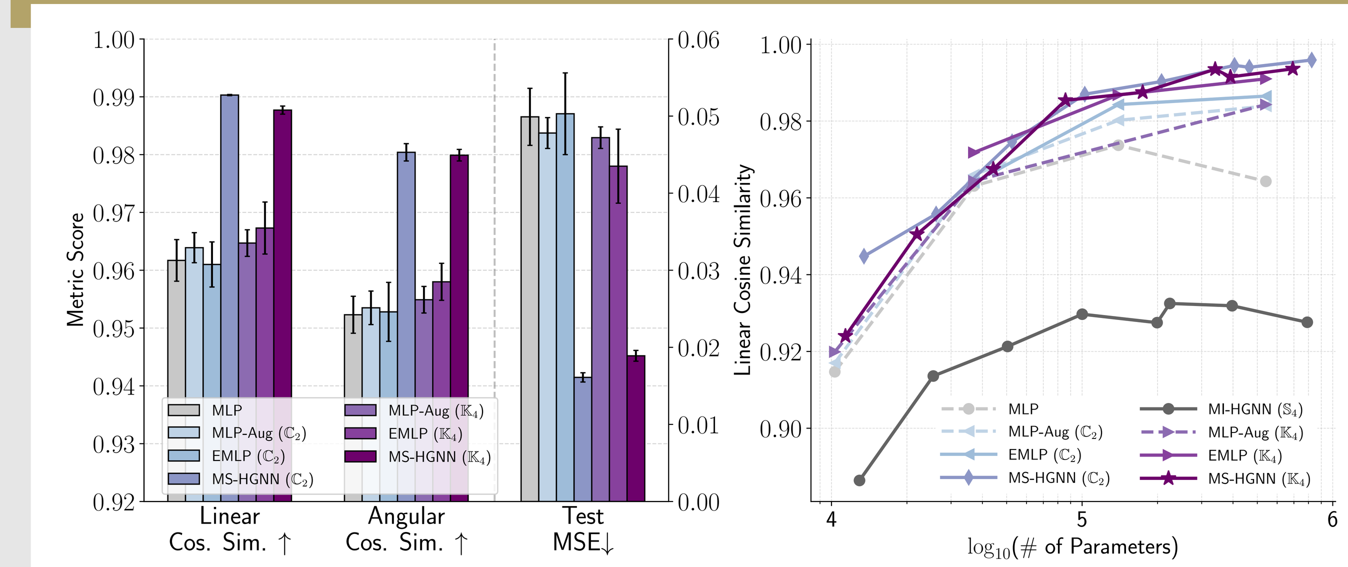

Figure 5: **Left:** Centroidal momentum estimation results and **Right:** Linear cosine similarity for models of varying sizes on the **synthetic Solo** dataset [3].

Task: Predict linear & angular momentum from joint-space inputs.

- MS-HGNN-$\mathbb{C}_2/\mathbb{K}_4$ outperform all baselines [3][2].
- MS-HGNN-$\mathbb{C}_2$ achieves ~0.945 Cos. Sim. with 13.5k params.

Conclusion: Embedding correct morphological symmetry enables accurate and compact momentum estimation; MI-HGNN fails due to symmetry mismatch.

Conclusion & Future Work

We propose MS-HGNN, a general framework for robotic dynamics learning that embeds kinematic structures and morphological symmetries into a graph-based neural architecture. By integrating symbolic priors, it combines the strengths of symbolic reasoning and neural networks. Theoretical and empirical results show improved generalization, sample efficiency, and interpretability. Future directions include integrating more physical priors, extending to meta- and reinforcement learning, and unifying perception and control.

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[2] D. Butterfield and et al., "MI-HGNN: Morphology-informed heterogeneous graph neural network for legged robot contact perception," in *Proc. IEEE Int. Conf. Robot. and Automation*, 2025.

[3] D. F. O. Apraez and et al., "On discrete symmetries of robotics systems: A group-theoretic and data-driven analysis," in *Proc. Robot.: Sci. Syst. Conf.*, 2023.